Vortex Induced Line Vibration

By Dave Lang, 6 Mar 2010

Introduction

When the wind-induced vortex shedding frequencies from lines are near the natural line vibration mode frequencies, then a condition termed "Lock-in" can occur in which resulting resonances can eventuate in operationally annoying oscillations of lines under tension. When these frequencies approach one-another to within even 20-30%, a Lock-in coalescence can occur. Therefore it would be prudent to know in advance whether such may become a problem with a projected atmospheric tether application. A very good example of such a situation might be a high-altitude, tethered, wind harvesting ~~system~~ rotary-kite system such as that proposed by Skymill Energy (with which I have some familiarity, and that presents a complex set of potential VIV environments as it progresses through its harvesting cycles).

This document identifies a method whereby a scheme such as Skymill can be examined to determine potential propensity for Vortex Induced Vibration (VIV). Such a scheme becomes quite complicated for a tethered system such as Skymill due to the large range of tether frequencies (ie. line tensions) and vortex shedding frequencies associated with normal operations.

To effect a VIV assessment of a system, basically, three things need to be accomplished at appropriate representative points of time during operations:

1. Reynold's number dependent "shedding frequencies" must be determined at points along the length of the tether,

2. Tether-line "Transverse natural frequencies" must be determined, and,

3. A method must be devised to display results so as to reflect any propensity for VIV.

Since Skymill operation is a dynamic process in which the combinations of the above identified parameters are dependent upon the specific operational trajectories being executed, these determinations become dependent upon the wind levels (at each altitude), and the tether line tensions being experienced as the trajectories unfold. For this reason it is logical to incorporate these analytical calculations into the GTOSS processing complex for simulations of Skymill, and then display these as interpretable results as each Skymill trajectory unfolds, so as to determine the potential for VIV of specific trajectories as well as "families of trajectories" representing characteristic situations Skymill may encounter".

While the above described frequency-based VIV assessment appears straight forward and highly deterministic, the actual prediction of VIV Lock-in is much less predictable, and historically has depended upon significant amounts of empirical findings related directly to the attributes of specific applications. The extent of system damping and "virtual accrued mass" (a fluid dynamic effect) all have a tendency to lower the natural frequencies of the lines, and/or mitigate VIV response.

The Vortex Shedding Frequencies.

The identification of Vortex Shedding Frequencies is based on the parameter called the Strouhal number, a dimensionless "modeling parameter" (similar to the dimensionless Reynolds parameter used to characterize flow regimes). Such modeling parameters can be very powerful in investigations since they can often render very general results and conclusions for a wide spectrum of dynamic regimes. Such is the case for the SkyMill application. The Strouhal number is defined as:

$$S = f_s \, {}^{d}\!/_{U}$$

Where:

S = Strouhal number
f_s = Voretx shedding frequency (Hz)
d = Cylinder diameter (ie SkyMill tether diameter)
U = Fluid cross-flow speed

So, in our situation, if one knows the Strouhal number for a particular flow regime, then the shedding frequency can be determined as a function of the line diameter and wind speed by inverting the above equation:

$$f_s = {}^{SU}\!/_{d}$$

As it turns out, such information is in fact known. Below is a graph defining the Strouhal number for a an *infinite cylinder* (ie. tether for our purposes) over a range of Reynolds number that encompasses the Reynolds number regime under which SkyMill might potentially be operating. The data was obtained from a lecture presented by Prof. Alexandra H. Techet of the MIT Mechanical Engineering Department. This is but one of many very similar Strouhal number tabulations to be found in the literature, all looking much the same. To use this chart, we must know the Reynold's number range related to SkyMill operation (see next section).

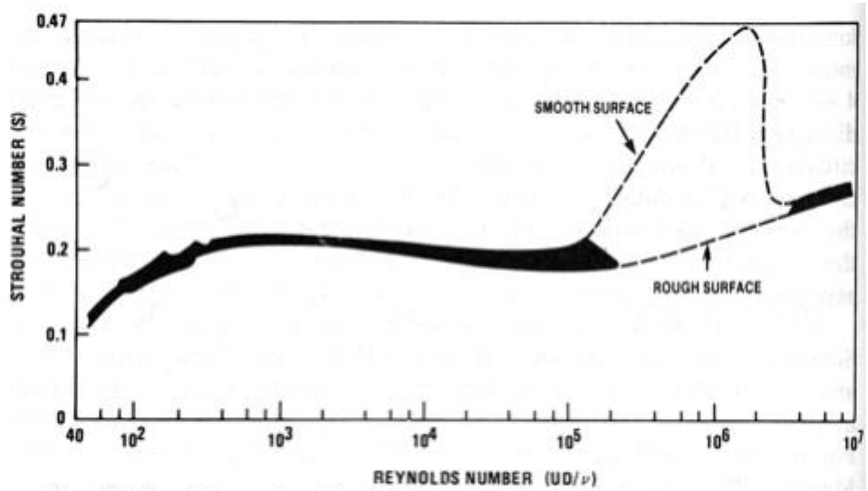


Fig. 3-3 Strouhal number–Reynolds number relationship for circular cylinders (Lienhard, 1966; Achenbach and Heinecke, 1981). $S \approx 0.21\ (1-21/Re)$ for $40 < Re < 200$ (Roshko, 1955).

The Reynold's Numbers.

The first step in assessing VIV is to gain an understanding of the range of Reynold's number that a Skymill might reasonably experience during operation. Reynold's number (R_E) for the Skymill tether would be defined as:

$$R_E = \frac{\rho U d}{\mu}$$

Where:

- ρ = Atmospheric density
- μ = Atmospheric "Dynamic Viscosity" (also called "Absolute Viscosity")
- d = Cylinder diameter (SkyMill tether diameter)
- U = Fluid cross-flow speed

The Atmospheric Properties.

The density is available as a standard atmospheric property (in GTOSS) as a function of altitude. The atmospheric viscosity is the viscosity of air, an attribute that varies as the temperature; the atmospheric temperature is available as a standard atmospheric property as a function of altitude also. The variation of air's viscosity as a function of temperature across the range of temperatures that Skymill will normally operate of -100 deg F (200 deg K) to +100 deg F (311 deg K) can be expressed by the relationship,

$$\mu = 2.78x10^{-7} + 0.0107x10^{-7}T_r$$

Where:

- μ = Dynamic Viscosity (lbf-sec/ft^2)
- T = Atmospheric temperature (K)
- T_r = Reduced temperature (K), = T - 200

Assessing SkyMill Reynold’s Number Range.

As a precursor to determining the Strouhal numbers that might be experienced along the SkyMill tether, it is necessary to examine the SkyMill operational flight envelope to determine the corresponding range of Reynold’s number that could be experienced.

By examination of the formula for Re, it is evident Re increases *proportionally* to line diameter, incident airspeed and density, and *inversely* with the viscosity. Now, the line diameter is a constant; over the range of temperatures expected in the SkyMill flight envelope, the viscosity is essentially also a constant (especially when compared with the large fluctuations in both airspeed and density).

So, for SkyMill, the following generalities can be noted:
- The higher the tether diameter, the higher the Re value.
- The higher the airspeed, the higher the Re value.
- The higher the altitude, the **lower** the Re value.

To further pin down this variation, a line diameter of **1.5 inches** will be chosen; Note, if line diameter is *doubled (or halved)*, then all the corresponding Re values will also be *doubled (or halved)*. Below is a table showing the Re values **(in Bold)** corresponding to a wide range of possible combinations of flight conditions for a SkyMill.

Table of SkyMill Reynold’s Number Range

		Altitude			
Airspeed		(km) (ft)	(km) (ft)	(km) (ft)	(km) (ft)
(knots)	(ft/sec)	1.5 5,000	4.6 15,000	7.6 25,000	10.7 35,000
10	17	**12,000**	**9,200**	**7,000**	**5,200**
20	34	**24,000**	**18,500**	**14,000**	**10,400**
40	67	**47,100**	**36,500**	**27,700**	**20,600**
60	101	**71,100**	**55,000**	**41,800**	**31,000**
80	135	**95,000**	**73,600**	**55.900**	**41,500**
100	169	**119,000**	**92,100**	**70,000**	**52,000**
140	236	**166,100**	**128,600**	**97,700**	**72,600**

Re values in this table, ranging from **5,000 to 166,000**, were constructed with no regard for correlation between airspeed and altitude (winds generally blow harder high aloft than near the ground, etc), thus it represents a wider range of Re value than might actually be encountered. If we also assume alternative lines of +/-50% diameter, this Re range then extends from **2,500 => 332,000**, a very comprehensive **Re** survey of SkyMill operations.

The Strouhal Numbers.

We can now re-visit the chart of Strouhal numbers to examine the range of Strouhal number that might be experienced by SkyMill. Below, the range of **Re** determined for SkyMill operations (in above table) has been superimposed on this chart.

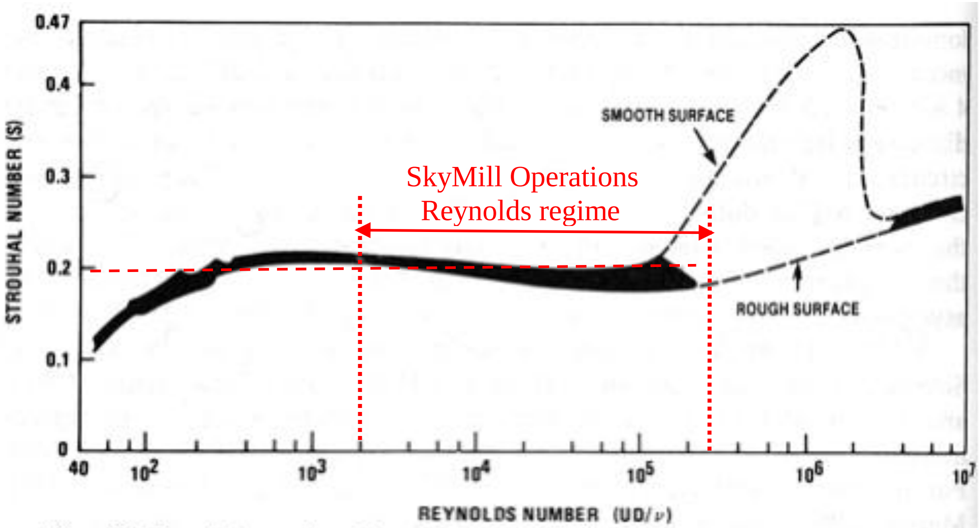


Fig. 3-3 Strouhal number–Reynolds number relationship for circular cylinders (Lienhard, 1966; Achenbach and Heinecke, 1981). $S \approx 0.21\ (1-21/Re)$ for $40 < Re < 200$ (Roshko, 1955).

From the above, it is clear that for practical purposes, an essentially constant value of Stouhal number of about **0.2** will characterize vortex shedding over the entire operational envelope of SkyMill. Thus, it is comparatively easy to construct the corresponding vortex shedding frequency attributes at various points along the SkyMill tether. Suppose that the winds of operational interest are assumed to range from 5 knots to 140 knots. At 5 knots, there is little concern regarding lock-in resonances because there is so little energy being delivered into the tether by the wind that material and non-linear aerodynamic damping effects should serve to limit any resonances that develop. For the selected wind speeds, a "shedding frequency" table representing all combinations of the nominal (green) and 2 alternate line diameters can be constructed:

Table of Typical Vortex Shedding Frequencies for SkyMill

Line dia (inch)	5 kt (Hz)	10 kt (Hz)	20 kt (Hz)	40 kt (Hz)	60 kt (Hz)	80 kt (Hz)	100 kt (Hz)	140 kt (Hz)
1.0	**20**	**41**	**82**	**163**	**245**	**326**	**408**	**565**
1.5	**14**	**27**	**55**	**107**	**163**	**214**	**270**	**376**
2.0	**10**	**20**	**41**	**82**	**119**	**163**	**201**	**283**

Note that the table above implies that a SkyMill tether line will generally be shedding vortices at different frequencies along its length; this is because the (altitude dependent) wind environment combines with tether dynamic state to induce varying relative winds. In effect, these vortices represent a multi-harmonic transverse-deflection forcing function along the length of the line and has the potential to engage in *lock-in resonance* with the natural line frequency harmonics. So to exhaustively portray the potential for VIV resonance, it will be necessary to represent the shedding frequencies that are occurring in appropriately spaced altitude bands of the tether line. Thus during the execution of a typical SkyMill power cycle trajectory, *at each point in time*, and at *each altitude band* at that point in time, there will correspond a value of vortex shedding frequency.

What remains to be seen now is how these shedding frequencies compare to the natural transverse line frequencies of a SkyMill tether. To do this we will need to evaluate the natural line frequencies of the SkyMill during its operational phases.

The Skymill Tether Natural Frequencies

The *transverse natural frequencies* exhibited by a tether depend upon the ***tension*** in the line in combination with line ***length*** and the ***lineal density***. This relationship is given by:

$$f_n^T = \frac{n}{2L}\sqrt{\frac{T}{\rho_L}}$$ n = 1, 2, 3, ….natural mode number

Where: $f^T{}_n$ = the *transverse* Natural Frequency (Hz), and,

T = Tension

L = Length

ρ_L = Lineal mass density

Thus for a given SkyMill tether length and tension, there is a single:

- 1st natural tether line frequency
- 2nd natural tether line frequency
- 3rd natural tether line frequency

. . . . etc

Each one of these frequency modes has associated with it a unique *transverse deflection shape* that manifests itself when the tether is vibrating in one of its "natural modes".

These natural modes (that vibrate at their corresponding natural frequencies) are susceptible to resonant lock-in with the shedding frequencies. An isolated *altitude region* of the tether may be shedding vortices at near the 1st natural tether frequency, and this has the potential to excite the entire line to 1st mode oscillations. However, of note, is the fact that the effectiveness of such a localized shedding phenomenon can be greatly diminished depending upon where spatially the phenomenon is occurring; for example if such a region of shedding was occurring (due to fortuitously favorable wind) at, say, the middle of the tether, then it would be far more effective in driving a resonant response of the 1st natural mode, than if the shedding were occurring near either end of the tether.

Expanding upon this explanation, the natural modes of vibration of the SkyMill tether are characterized by unique **node points** along their length (ie. points that do no deflect during natural vibration) and **anti-nodes** (points that experience maximum excursions) when a tether is undergoing a pure modal oscillation. Thus vortex shedding at a natural mode frequency that occurs spatially near a *node* is least effectual in exciting the tether, while that same shedding if it were to occur at an *anti-node* would have maximum effect.

Combining Vortex Shedding frequencies and the Natural Tether frequencies.

Ideally these shedding frequencies and natural tether frequencies are combined into a composite presentation in which the tether natural frequencies can be compared to the vortex shedding frequencies to assess margins against "lock-in" and generally visualize the VIV situation. Due to the dynamic nature of the SkyMill, a broad range of conditions can present themselves in the form of varying tether lengths (as line is paid-out and retrieved during the power stroke and return stroke), and varying and transient tension states; this will create a large and transient universe of tether natural frequencies that unfold as a function of time. **Note that the natural tether frequencies manifest themselves as integral multiples of the fundamental (ie. 1st mode) frequency at each point in time (corresponding to a particular length and tension condition).**

Then there are *shedding frequency* variations that exist along the length of the line as different portions of tether become subject to varying induced transient relative wind states. Thus, to fully assess a VIV situation for SkyMill, we must examine families of operational envelopes corresponding to an assortment of trajectories spanning different altitude regimes and wind conditions. For this reason, the assessment of VIV propensity will be presented as a time varying graph unique to a particular trajectory. While this appears daunting at first, it will likely develop that certain characteristics of the SkyMill operational trajectories will prove to make VIV either unlikely, or if found to be problematic, will dictate certain changes in the operational trajectory approaches.

Thus, the results of a SkyMill GTOSS simulation run now include this presentation. Due to the dynamic nature of the trajectories, such presentations will be unique to each case, appearing more less different for each unique set of operational conditions. Eventually enough knowledge will be gained regarding the VIV phenomenon to either exonerate SkyMill from concern, or, potentially escalate the VIV examinations, or, at worst, to bring about operational trajectory changes.

Preliminary Assessment of SkyMill VIV Situation

As a preliminary assessment of the SkyMill VIV situation 3 cases will be examined:

1. Harvesting in a generally windy environment (65 knots aloft, diminishing to about 15 knots at the ground).

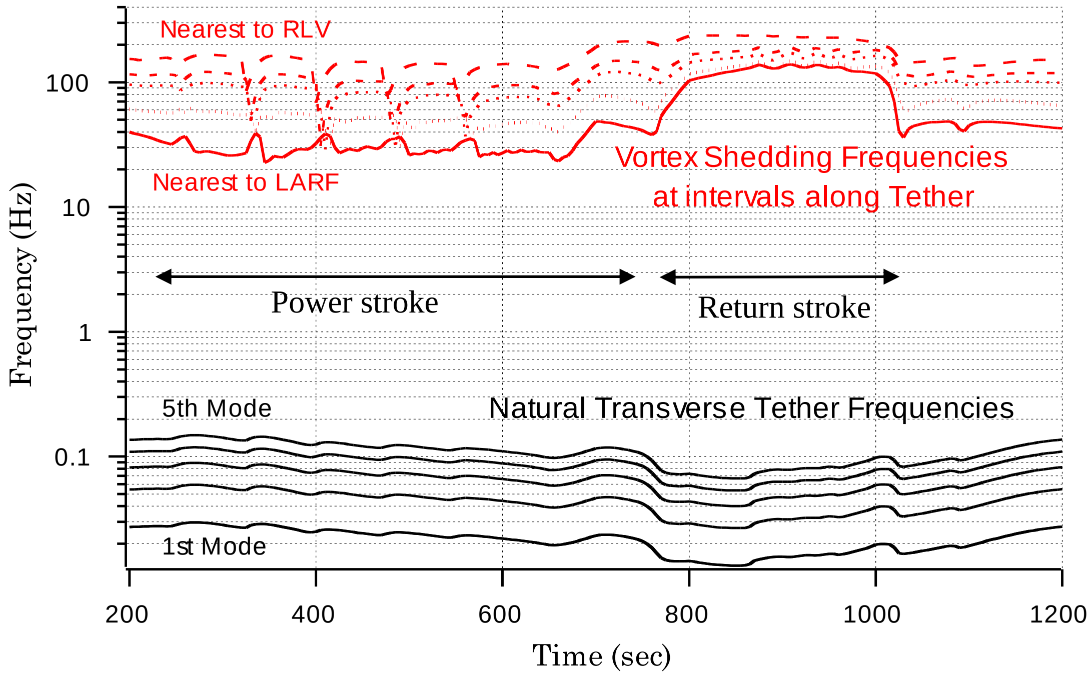
2. Harvesting in "jet stream" (75 knots aloft, diminishing to essentially no-wind at the ground).

3. A low altitude/low wind speed launch/climb-out.

This should provide us with a first insight into the nature of VIV and SkyMill.

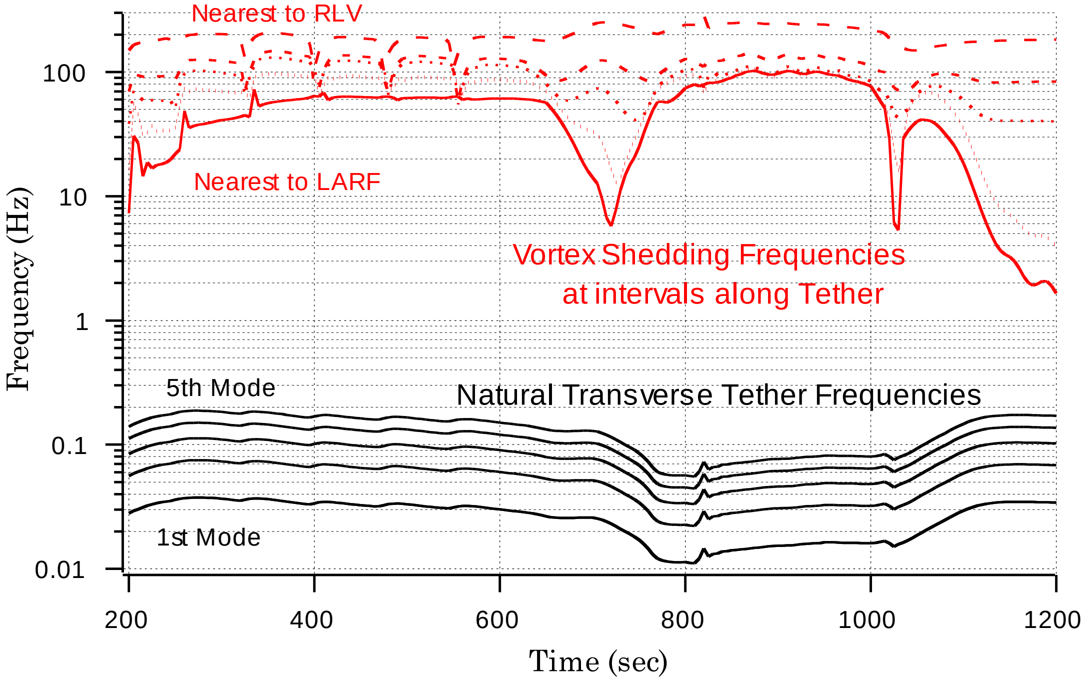
These cases (except for the climb-out case) correspond to a 100 ft rotor, using a 1.5 inch Spectra line operating in the 30,000 ft altitude range, representing one full execution of the operational power-harvesting trajectory.

Case 1: Operational SkyMill VIV Situation (Generally High Wind Day)



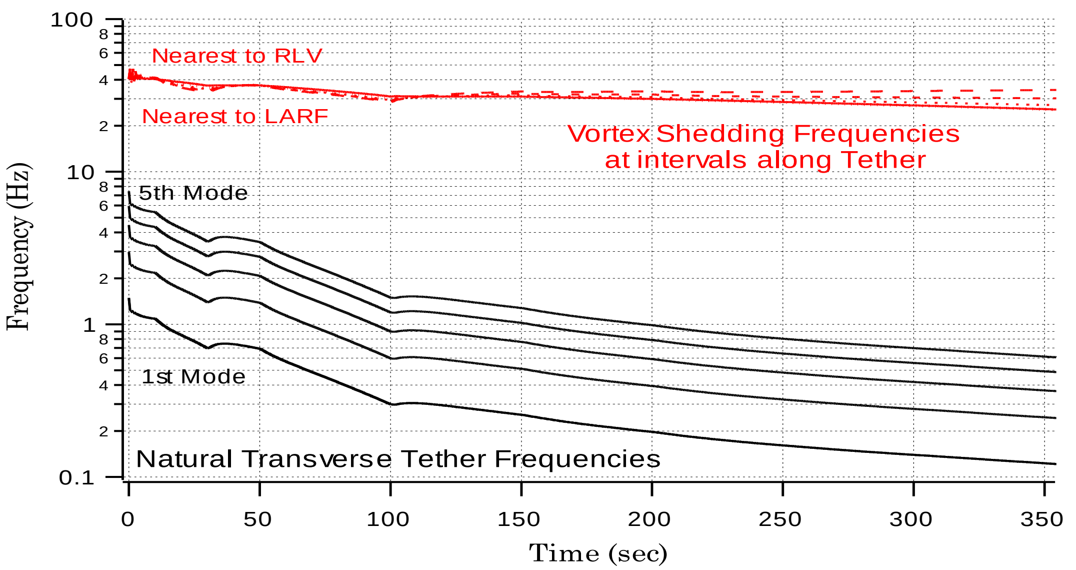
In this case, wind varies from 65 kt peak, down to about 15 kt at the ground. This SkyMill produces a nominal 1 mW of average power. The graph above, for each instant of time, shows the vortex shedding frequencies evaluated at 5 uniform intervals along the tether from the LARF to the RLV. Also superimposed on this graph, for each instant of time, are the tether's first 5 natural transverse frequencies. Note that the various trajectory phases (such a the power-stroke, return stroke, etc), manifest themselves in both the shedding frequencies and the line natural frequencies. Ideally, (to be indicative of no VIV propensity) these two families of frequencies will be separated widely. When families of frequencies get within about 20-30% of each other, then the threat of VIV emerges as a possibility. ***It is clear from examining this graph that the shedding frequencies are staying well clear of the line frequencies, which of course is good!***

Case 2: Operational SkyMill VIV Situation (Harvesting a Jet Stream)



In this case, wind varies from 75 kt peak, down to essentially zero at the ground. This SkyMill produces a nominal 2 mW of average power. This case is more extreme than the previous case, and shows a wide variation in the induced relative wind state on the line. The closest that the shedding frequencies approach the line frequencies is at 1200 sec. Here the lowest shedding frequency is about 2 Hz, while the nearest line frequency is about 0.2 Hz, so there is still a factor of 10 between them, thus indicating virtually no propensity for lock-in of the frequencies.

Case 3: Operational SkyMill VIV Situation (Ground Climb-out in Minimal Wind)



Here, SkyMill RLV is hovering 300 ft off the ground in a 15 kt wind. Line is slowly paid out, allowing the RLV to climb, then increasingly faster into a slowly building wind environment, reaching just over 2,000 ft altitude by 350 sec. Note: the natural line frequencies are quite a bit higher (than previous cases) due to the short initial tether length, but immediately drop in value as the line pays out (at essentially constant tension). Again, it is evident that there is virtually no indication of a potential VIV issue in this particular phase of operation.

A Note on Higher (multiple) Mode VIV

It is <u>not</u> clear from the literature that VIV higher mode (or multiple mode) Lock-in is currently well understood, or predictable. There are complicating issues involving attributes such as coherency (correlation) of the shedding forcing functions with the spatial-node distributions of the higher modes. “Spatial Correlation” (being conducive to Lock-in) is diminished by the presence of random turbulence. For SkyMill, the spatial correlation issue is particularly intriguing since wind speed -vs- altitude variation could play a major role in either diminishing or enhancing spatial correlation There are other effects involving the “capture bandwidth” which is even less well understood for higher mode Lock-in. Likely the one conclusion that might be drawn is that it would be more difficult for vortex shedding to excite higher modes than lower modes.

Conclusion

Based on these limited results to this point, it appears that VIV may not be a problem with SkyMill, however, as design proceeds, vigilance will be exercised to detect any propensity for VIV; this new GTOSS analysis capability will facilitate our due diligence.